Mid-Semestral Exam
Algebra-IV
B. Math - Second year
2016-2017

Time: 3 hrs
Max score: 100

Answer question (**1**) and any (**4**) from the rest. Throughout, F shall denote a field.

(1) State true or false. Justify your answers. No marks will be awarded in the absence of proper justification.

(a) A regular 9-gon is constructible.
(b) There exists finite algebraically closed fields.
(c) The polynomial $x^4 + 3x + 3$ is irreducible over $\mathbb{Q}(\sqrt[3]{2})$.
(d) If ζ_n is a primitive nth root of unity, then $\zeta_5 \notin \mathbb{Q}(\zeta_7)$.
(e) For any $n \geq 1$, there exists an irreducible polynomial of degree n in $\mathbb{F}_p[x]$. (8×5)

(2) (a) Show that the splitting field of a polynomial of degree n over a field F is of degree atmost $n!$ over F.
(b) Show that the polynomial $x^p - 2$ is irreducible over $\mathbb{Q}(\zeta_p)$, where ζ_p is a primitive pth root of unity. (7+8)

(3) Find the splitting field and the degree of the splitting field of the polynomial $f(x) = x^6 + 1$
(a) over $\mathbb{Q}$, and
(b) over $\mathbb{F}_2$. (7+8)

(4) Let p be a prime, and n be a positive integer. Then show that,
(a) there exists a field of order p^n,
(b) any two fields of order p^n are isomorphic. (8+7)

(5) Consider the polynomial $f(x) = x^n - 1$ over a field F. Show that
(a) if $char(F) = 0$ or p, with $p \nmid n$, then $f(x)$ is separable over F.
(b) if $char(F) = p$, and $n = p^k m$, $p \nmid m$, then $f(x)$ has precisely m distinct roots over F, each of multiplicity p^k. (5+10)